

New constraints on the 3-3-1 model with right-handed neutrinos

B. L. Sánchez–Vega^{1,*}, E. R. Schmitz^{2,†} and J. C. Montero^{1,*}

¹ *Universidade Estadual Paulista (Unesp), Instituto de Física Teórica (IFT), São Paulo*

R. Dr. Bento Teobaldo Ferraz 271, Barra Funda, São Paulo - SP, 01140-070, Brazil

² *Bethe Center for Theoretical Physics and Physikalisches Institut,*

Universität Bonn, Nussallee 12, D-53115 Bonn, Germany.

Abstract

In the framework of a $3 - 3 - 1$ model with right-handed neutrinos and three scalar triplets we consider different spontaneous symmetry breaking patterns seeking for a non-linear realization of accidental symmetries of the model, which will produce physical Nambu-Goldstone (NG) bosons in the neutral scalar spectrum. We make a detailed study of the safety of the model concerning the NG boson emission in energy loss processes which could affect the standard evolution of astrophysical objects. We consider the model with a \mathbb{Z}_2 symmetry, conventionally used in the literature, finding that in all of the symmetry breaking patterns the model is excluded. Additionally, looking for solutions for that problem, we introduce soft \mathbb{Z}_2 -breaking terms in the scalar potential in order to remove the extra accidental symmetries and at the same time maintain the model as simple as possible. We find that there is only one soft \mathbb{Z}_2 -violating term that can get rid of the problematic NG bosons.

*brucesan@ift.unesp.br

†ernany@th.physik.uni-bonn.de

*montero@ift.unesp.br

I. INTRODUCTION

Recently experiments have reached the capability of exploring the TeV energy scale and the Standard Model (SM) still has an impressive accordance with data. However, there are at least two important aspects which put in evidence the incompleteness of the SM and, hence, the need for new physics. Namely they are i) the non-zero neutrino masses, which allows for neutrino flavor oscillation during space propagation, and ii) the lack of a consistent candidate for dark matter (DM), assuming that DM is a manifestation of an unknown particle.

In this way, models based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ gauge symmetry (the so called $3-3-1$ models, for shortness) [1–4], are interesting extensions of the SM. Since in these models the electroweak interaction is supposed to be invariant under transformations of a larger gauge group, the matter content can be chosen to accommodate new appropriate degrees of freedom in order to implement phenomenologically attractive features, as generation of neutrino masses, for instance.

The quark sector will also be enlarged and new quarks will be present, with the possibility of possessing exotic electric charges. Chiral anomaly cancellation is assured provided we have the same number of triplets and anti-triplets, including color counting. If we assume that there is a symmetry between leptons and quarks in such a way that the number of families of leptons is equal to the number of families of quarks, say N_f , then, we must have $N_{\text{anti}} = 2N_f/3$ quark families transforming under the $\bar{\mathbf{3}}$ representation of the $SU(3)_L$ group, and the other $N_f - N_{\text{anti}}$ families transforming under the $\mathbf{3}$ one. It means that the number of families must be three or a multiple of three. As a consequence, differently from the SM, the model is anomaly free only when the total number of families is considered.

Moreover, if we bring in the QCD asymptotic freedom, the number of families must be just 3. The renormalization group β function, which gives the behavior of the strong coupling with the transferred momentum, can be computed in perturbation theory and at one-loop level its sign is governed by the factor $-(11 - \frac{2}{3}n_q)$, where n_q is the number of quark flavors (which is 6 in the SM). To keep the negative sign, the only possibility compatible with asymptotic freedom, n_q must be ≤ 16 . In $3-3-1$ models with $N_f = 3$ we have 9 quark flavors and the sign of the β function remains correct. However, for $N_f = 6$ there are 18 quark flavors and the β function gets the wrong sign so that the number of families must

be 3. It is interesting to note that the anomaly cancellation relates the number of families to the number of colors. In this sense the $3 - 3 - 1$ models shed some light on the family replication problem.

Besides this particular feature, $3 - 3 - 1$ models are promising alternatives to the SM for they present a variety of interesting properties. Among them we can mention: i) in the model described in Ref. [1] we find that the $U(1)_N$ and the $SU(3)_L$ coupling constants, g_N and g_L , respectively, obey the relation $t^2 = (g_N/g_L)^2 = \sin^2 \theta_W / (1 - 4 \sin^2 \theta_W)$. It means that there is a Landau-like pole at an $\mathcal{O}(\text{TeV})$ energy scale μ such that $\sin^2 \theta_W(\mu) = 1/4$ [5], and hence it explains why $\sin^2 \theta_W(\mu) < 1/4$ is observed; ii) the electric charge quantization is independent of the nature of neutrinos, i.e., regardless if they are Majorana or Dirac fermions [6]; iii) the Peccei-Quinn symmetry, needed to solve the strong CP problem, is almost automatic in these models [7, 8].

For each $3 - 3 - 1$ model, depending on the matter content accommodated in the $SU(3)_L$ triplets and singlets, an appropriate scalar sector has to be introduced. As usual, the Yukawa terms are responsible for generating mass to the matter fields and also for matter fields-(pseudo-)scalar interactions. Therefore, the $3 - 3 - 1$ scalar sector is richer than that of the SM, and this fact can be explored to give explanations to some phenomenological aspects that do not have a consistent answer or are out of the scope of the SM framework. Among others, some aspects closely related to the scalar sector and the Yukawa interactions are i) a mechanism for generating tiny neutrino masses [9–11], ii) a natural explanation for the fermion mass hierarchy, and iii) a consistent DM candidate [9, 12–14].

In order to achieve these goals, and recover the low energy physics, spontaneous symmetry breaking (SSB) must occur. For each linearly independent broken generator, there will be a Nambu-Goldstone (NG) boson, if the number of broken symmetries exceeds the number of massive gauge bosons, this implies physical massless NG bosons which are potentially dangerous since they couple to fermions and could in principle escape from star nuclei carrying out energy, thus modifying the standard evolution of these objects. Their interactions with nucleons [15–21] and electrons [15, 22–25], are parameterized by g_{nnJ} and g_{eeJ} , respectively, (where J means the physical NG boson), are bounded by the standard evolution of neutron stars and supernovae; red giants, super giants, sun, white dwarfs and neutron star crusts, and can constrain the parameters of a given considered scenario or even rule it out. This is the main goal of this work.

This work is organized as follows. In the Sec. II, we present the general features of the $3-3-1$ model, including its matter content, Yukawa and scalar potential. In the Sec. III, we consider the model with a \mathbb{Z}_2 symmetry widely used, and present the consequences of this choice, such as the number and form of physical NG bosons for each display of vacuum expectation values (VEVs), and a discussion of the constraints from $g_{e\bar{e}J}$ and Z invisible decay on them. Finally, in the Sec. IV, we add soft \mathbb{Z}_2 -breaking terms to the previous scalar potential and analyze their consequences using as a guidance the constraints from the $g_{e\bar{e}J}$ and g_{nnJ} couplings, and also the W mass and the ρ parameter values; a discussion of the constraints at a symmetry point of view is also presented. Sec. V is devoted to our conclusions.

II. THE MODEL

The $3-3-1$ model with right-handed neutrinos considered in this paper was proposed in Ref. [10] and it has been subsequently considered in Refs. [8, 9, 12, 26–30], where different aspects of this model were studied. This model shares appealing features with other versions of $3-3-1$ models [1–3, 31–35]. It offers for example an explanation to the number of the fermion families and makes easily the implementation of the Peccei-Quinn mechanism [8]. Furthermore, right-handed neutrinos are in the same multiplet of the SM leptons, which allows terms of mass for the neutrinos at tree level, although the smallness of the those masses remains unexplained.

Generally speaking, this model is based on the gauge symmetry group $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$, where C stands for color and L for left chirality, as in the SM; and N stands for a new charge different than the SM hypercharge Y . The N values are assigned in order to obtain the SM hypercharge $Y = 2N \mathbf{1}_{3 \times 3} - \frac{1}{\sqrt{3}} T_8$, after the first spontaneous symmetry breaking, and as a consequence the electric charge $Q = T_3 + \frac{Y}{2} \mathbf{1}_{3 \times 3}$, where T_3, T_8 are the diagonal $SU(3)_L$ generators, whereas $T_9 (= \frac{N}{2} \mathbf{1}_{3 \times 3})$ is the generator of the $U(1)_N$ group. Symmetry breaking and fermion masses are achieved with at least three $SU(3)_L$ triplets, η, ρ, χ , as shown in Ref. [30]. These triplets are in the $(1, \mathbf{3}, -1/3)$, $(1, \mathbf{3}, 2/3)$ and $(1, \mathbf{3}, -1/3)$ representations of the $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ symmetry groups, respectively. In more detail, the scalar triplets are expressed by

$$\eta = \left(\eta_1^0, \eta_2^-, \eta_3^0 \right)^T, \quad \rho = \left(\rho_1^+, \rho_2^0, \rho_3^+ \right)^T, \quad \chi = \left(\chi_1^0, \chi_2^-, \chi_3^0 \right)^T.$$

The fermionic content of the model is richer than the SM because the fields are embedded into non-trivial representations of a larger group, $SU(3)_L$. The left-handed fields belong to the following representations

$$\begin{aligned} \text{Leptons: } f_{aL} &= \left(\begin{array}{ccc} \nu_a & e_a & N_a^c \end{array} \right)_L^T \sim (1, \mathbf{3}, -1/3), \\ \text{Quarks: } Q_L &= \left(\begin{array}{ccc} u_1 & d_1 & u_4 \end{array} \right)_L^T \sim (\mathbf{3}, \mathbf{3}, 1/3), \\ Q_{bL} &= \left(\begin{array}{ccc} d_b & u_b & d_{b+2} \end{array} \right)_L^T \sim (\mathbf{3}, \bar{\mathbf{3}}, 0), \end{aligned} \quad (1)$$

where $a = 1, 2, 3$ and $b = 2, 3$; and “ \sim ” means the transformation properties under the local symmetry group. Notice that N_a stands for the right-handed neutrinos. Additionally, in the right-handed field sector we have

$$\text{Leptons: } e_{aR} \sim (1, 1, -1), \quad (2)$$

$$\text{Quarks: } u_{sR} \sim (3, 1, 2/3), \quad d_{tR} \sim (3, 1, -1/3), \quad (3)$$

with a in the same range as in the previous case; $s = 1, \dots, 4$ and $t = 1, \dots, 5$.

Regarding the Yukawa Lagrangian, we can write it as follows

$$\mathcal{L}_{\text{Yuk}} = \mathcal{L}_{\text{Yuk}}^\rho + \mathcal{L}_{\text{Yuk}}^\eta + \mathcal{L}_{\text{Yuk}}^\chi, \quad (4)$$

with

$$\begin{aligned} \mathcal{L}_{\text{Yuk}}^\rho &= \alpha_t \bar{Q}_L d_{tR} \rho + \alpha_{bs} \bar{Q}_{bL} u_{sR} \rho^* + Y_{aa'} \epsilon_{ijk} (\bar{f}_{aL})_i (f_{a'L})_j^c (\rho^*)_k + Y'_{aa'} \bar{f}_{aL} e_{a'R} \rho \\ &\quad + \text{H.c.}, \end{aligned} \quad (5)$$

$$\mathcal{L}_{\text{Yuk}}^\eta = \beta_s \bar{Q}_L u_{sR} \eta + \beta_{bt} \bar{Q}_{bL} d_{tR} \eta^* + \text{H.c.}, \quad (6)$$

$$\mathcal{L}_{\text{Yuk}}^\chi = \gamma_s \bar{Q}_L u_{sR} \chi + \gamma_{bt} \bar{Q}_{bL} d_{tR} \chi^* + \text{H.c.}, \quad (7)$$

where $a', i, j, k = 1, 2, 3$ and with a, b, s, t in the same range as in the previous case. From Eqs. (5-7) it can be seen that, in general, flavor-changing neutral currents (FCNCs) can be induced because the quark fields interact with different neutral scalar fields simultaneously. This characteristic is shared by most of multi-Higgs models [36]. However, some model dependent strategies to successfully overcome that problem have been proposed [3, 36, 37]. Among those, we can mention, for instance, choosing an appropriate direction in the VEV space, resorting to heavy scalars and/or small mixing angles in the quark and the scalar sectors, and considering adequate Yukawa coupling matrix textures. In particular, in this

model the non-SM quarks have the same electric charge as the SM ones. That means that these can mix with the latter ones and hence also induce FCNC. Despite that, this kind of FCNC is suppressed when the VEV which mainly controls the exotic quark masses is taken much larger than the electroweak mass scale [37]. Also, FCNC occurs in models which have an extra neutral vector boson. These can be handled in a similar way. See, for example, [38, 39]. Finally, we remark that from Eq. (5) it is clear that the lepton sector of the model is not afflicted by FCNC.

We also have that the most general scalar potential consistent with gauge invariance and renormalizability is given by

$$V(\eta, \rho, \chi) = V_{\mathbb{Z}_2}(\eta, \rho, \chi) + V_{\cancel{\mathbb{Z}_2}}(\eta, \rho, \chi); \quad (8)$$

with

$$\begin{aligned} V_{\mathbb{Z}_2}(\eta, \rho, \chi) = & -\mu_1^2 \eta^\dagger \eta - \mu_2^2 \rho^\dagger \rho - \mu_3^2 \chi^\dagger \chi \\ & + \lambda_1 (\eta^\dagger \eta)^2 + \lambda_2 (\rho^\dagger \rho)^2 + \lambda_3 (\chi^\dagger \chi)^2 + \lambda_4 (\chi^\dagger \chi) (\eta^\dagger \eta) \\ & + \lambda_5 (\chi^\dagger \chi) (\rho^\dagger \rho) + \lambda_6 (\eta^\dagger \eta) (\rho^\dagger \rho) + \lambda_7 (\chi^\dagger \eta) (\eta^\dagger \chi) \\ & + \lambda_8 (\chi^\dagger \rho) (\rho^\dagger \chi) + \lambda_9 (\eta^\dagger \rho) (\rho^\dagger \eta) + [\lambda_{10} (\chi^\dagger \eta)^2 + \text{H.c.}]; \end{aligned} \quad (9)$$

$$\begin{aligned} V_{\cancel{\mathbb{Z}_2}}(\eta, \rho, \chi) = & -\mu_4^2 \chi^\dagger \eta \\ & + \lambda_{11} (\chi^\dagger \eta) (\eta^\dagger \eta) + \lambda_{12} (\chi^\dagger \eta) (\chi^\dagger \chi) + \lambda_{13} (\chi^\dagger \eta) (\rho^\dagger \rho) \\ & + \lambda_{14} (\chi^\dagger \rho) (\rho^\dagger \eta) + \frac{f}{\sqrt{2}} \epsilon_{ijk} \eta_i \rho_j \chi_k + \text{H.c.} \end{aligned} \quad (10)$$

We have divided the total scalar potential $V(\eta, \rho, \chi)$ in two pieces, $V_{\mathbb{Z}_2}(\eta, \rho, \chi)$ and $V_{\cancel{\mathbb{Z}_2}}(\eta, \rho, \chi)$, for future convenience. The first part is invariant under a \mathbb{Z}_2 discrete symmetry ($\chi \rightarrow -\chi$, $u_{4R} \rightarrow -u_{4R}$, $d_{(4,5)R} \rightarrow -d_{(4,5)R}$) in contrast to the second one which is not. The model with such \mathbb{Z}_2 symmetry will be studied in detail in the next section.

The minimal vacuum structure in order to give masses for all the fields in the model is

$$\langle \rho \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & v_{\rho_2} & 0 \end{pmatrix}^T, \quad \langle \eta \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{\eta_1} & 0 & 0 \end{pmatrix}^T, \quad \langle \chi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & v_{\chi_3} \end{pmatrix}^T.$$

Specifically, the symmetry breaking pattern is done in two stages. First, when χ gains a VEV, $\langle \chi \rangle$, the exotic quarks gain masses and the symmetry, $\text{SU}(3)_C \otimes \text{SU}(3)_L \otimes \text{U}(1)_N$, is broken down to $\text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y$. After that, the VEV $\langle \rho \rangle$ gives mass to the three charged leptons and to two of the neutrinos [4, 10, 11]. Also, two up-type quarks and one

down-type quark gain masses from $\langle \rho \rangle$. Finally, $\langle \eta \rangle$ gives mass for the remaining quarks. In this last stage ($\langle \rho \rangle$ and $\langle \eta \rangle$ different from zero) the symmetry, $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, is broken down to $U(1)_Q$. Note that, although the η and χ scalar triplets are in the same representation of the gauge symmetries, we have defined, without loss of generality, the χ triplet as the one responsible for the first symmetry breaking stage. In other words, it is assumed that $\langle \chi \rangle > \langle \rho \rangle, \langle \eta \rangle$.

III. MODEL WITH \mathbb{Z}_2 SYMMETRY

It is a common practice to impose a discrete \mathbb{Z}_2 symmetry given by: $\chi \rightarrow -\chi$, $u_{4R} \rightarrow -u_{4R}$, $d_{(4,5)R} \rightarrow -d_{(4,5)R}$ and all the other fields being even under \mathbb{Z}_2 [4, 8–10, 12, 27, 28, 40]. This symmetry brings simplicity to the model allowing, for instance, to interpret the χ scalar as the responsible for the first step in the symmetry breaking pattern and in some sense, to mitigate the FCNC issues. Besides that, it also largely simplifies the scalar potential. In this scenario, the Yukawa Lagrangian interactions given in Eqs. (5-7) are slightly modified to

$$\mathcal{L}_{\text{Yuk}}^\rho = \alpha_a \bar{Q}_L d_{aR} \rho + \alpha_{ba} \bar{Q}_{bL} u_{aR} \rho^* + Y_{aa'} \epsilon_{ijk} (\bar{f}_{aL})_i (f_{bL})_j^c (\rho^*)_k + Y'_{aa'} \bar{f}_{aL} e_{a'R} \rho + \text{H.c.}, \quad (11)$$

$$\mathcal{L}_{\text{Yuk}}^\eta = \beta_a \bar{Q}_L u_{aR} \eta + \beta_{ba} \bar{Q}_{bL} d_{aR} \eta^* + \text{H.c.}, \quad (12)$$

$$\mathcal{L}_{\text{Yuk}}^\chi = \gamma_4 \bar{Q}_L u_{4R} \chi + \gamma_{b(b+2)} \bar{Q}_{bL} d_{(b+2)R} \chi^* + \text{H.c.}. \quad (13)$$

Furthermore, the \mathbb{Z}_2 symmetry forbids the terms in $V_{\mathbb{Z}_2}(\eta, \rho, \chi)$ to appear in the scalar potential. It implies that the model has actually a larger symmetry group. Specifically, we show in Table I all the $U(1)$ symmetries that the model really has (the global and local ones). Note there are two extra global symmetries, the baryonic one, $U(1)_B$, that remains unbroken, and the $U(1)_{\text{PQ}}$. The last one being a Peccei-Quinn like symmetry because it is anomalous in the color group. We also remark that these are symmetries of the entire Lagrangian.

In the minimal case, when only three VEVs - v_{η_1}, v_{ρ_2} and v_{χ_3} - are different from zero, the scalar sector has a Nambu-Goldstone (NG) boson, J , in the physical spectrum. It is given by

$$J = \frac{1}{N_J} \left(\frac{v_{\eta_1} v_{\chi_3}}{v_{\rho_2}} \text{Im} \rho_2^0 + v_{\chi_3} \text{Im} \eta_1^0 + v_{\eta_1} \text{Im} \chi_3^0 \right), \quad (14)$$

	Q_L	Q_{iL}	(u_{aR}, u_{4R})	$(d_{aR}, d_{(4,5)R})$	f_{aL}	e_{aR}	η	ρ	χ
$U(1)_N$	1/3	0	2/3	-1/3	-1/3	-1	-1/3	2/3	-1/3
$U(1)_B$	1/3	1/3	1/3	1/3	0	0	0	0	0
$U(1)_{PQ}$	1	-1	0	0	-1/2	-3/2	1	1	1

Table I: The $U(1)$ symmetries in the model when the \mathbb{Z}_2 discrete symmetry is considered.

with $N_J \equiv \left(v_{\eta_1}^2 v_{\chi_3}^2 v_{\rho_2}^{-2} + v_{\eta_1}^2 + v_{\chi_3}^2 \right)^{1/2}$. We emphasize that J in Eq. (14) is orthogonal to the NG bosons which are absorbed by the gauge vector bosons, as it should be. We have followed the method described in Refs. [41, 42] to accomplish that.

From the explicit form of J in Eq. (14), and the Lagrangian in Eq. (11), it is straightforward to calculate the coupling of J with the electron and positron, $g_{e\bar{e}J}$. It is explicitly given by

$$g_{e\bar{e}J} = \frac{\sqrt{2}m_e v_{\eta_1} v_{\chi_3}}{N_J v_{\rho_2}^2}, \quad (15)$$

where m_e is the electron mass. The existence of this coupling opens a new channel of energy loss in stars through the Compton-type process $\gamma + e^- \rightarrow e^- + J$. From the evolution of red-giant stars we have that [15, 22–25]

$$|g_{e\bar{e}J}| \lesssim g_{\max} \equiv 10^{-13}. \quad (16)$$

In order to impose a bound on the VEVs, we bring another piece of information. The mass of the W^\pm bosons is

$$M_{W^\pm}^2 = \frac{g_L^2}{4} \left(v_{\eta_1}^2 + v_{\rho_2}^2 \right), \quad (17)$$

where g_L is the gauge coupling constant of the $SU(3)_L$ group. Thus, $v_{\eta_1}^2 + v_{\rho_2}^2 = v_{\text{SM}}^2 \simeq 246^2 \text{ GeV}^2$ in order to obtain the SM W^\pm mass [43]. From Eqs. (15-17) the following upper bound on v_{χ_3} can be found

$$v_{\chi_3} \leq v_{\chi_{\max}}(v_{\rho_2}) \equiv v_{\rho_2} \left[2g_{\max}^{-2} m_e^2 / v_{\rho_2}^2 - 1 / \left(1 - v_{\rho_2}^2 / v_{\text{SM}}^2 \right) \right]^{-1/2}. \quad (18)$$

Thus, the upper bound on v_{χ_3} is a function of v_{ρ_2} . Because of Eq. (18), we can estimate that the largest value that v_{χ_3} can take is $v_{\chi_{\max}}(v_{\rho_2} \rightarrow v_{\text{SM}}^-) \simeq 11.5 \text{ keV}$. However, it is in

contradiction with the general assumption in this model which claims that $\langle \chi \rangle > \langle \rho \rangle, \langle \eta \rangle$. In addition to that, there is a fact that rules out this scenario with the \mathbb{Z}_2 symmetry. In order to understand it, notice that the NG boson in Eq. (14), that results from the breaking of the symmetries, is actually an axion because of the $U(1)_{PQ}$ symmetry in Table I. Moreover, the decay constant f_a for the axion is, in this case, given by

$$f_a = N_J. \quad (19)$$

From this and the upper bound in Eq. (18) we can find an upper bound on f_a as follows

$$f_a \leq f_{a\max}(v_{\rho_2}) \equiv v_{\text{SM}} \left[1 - \frac{v_{\rho_2}^2}{v_{\text{SM}}^2} + \frac{v_{\chi\max}^2(v_{\rho_2})}{v_{\rho_2}^2} \right]^{1/2}. \quad (20)$$

From the equation above, we can see that $v_{\chi\max}(v_{\text{SM}}) \approx 11.5 \text{ keV} \leq f_a \leq v_{\text{SM}}$. However, an axion with this small decay constant was ruled out long ago [44, 45].

In the symmetry breaking patterns with more than three VEVs different from zero, i.e., $(v_{\eta_1}, v_{\eta_3}, v_{\rho_2}, v_{\chi_3})$, $(v_{\eta_1}, v_{\rho_2}, v_{\chi_1}, v_{\chi_3})$, and $(v_{\eta_1}, v_{\eta_3}, v_{\rho_2}, v_{\chi_1}, v_{\chi_3})$, the situation is even worse. Although the exact form of the CP -even and CP -odd scalars present in the physical spectra depends on the particular breaking pattern of the symmetry, there are some general features that we can mention. In all cases, there are two physical NG bosons, J_I and J_R , the first in the CP -odd sector, and the second in the CP -even sector. One of them couples with the electron and positron, i.e., $g_{e\bar{e}J_I} \neq 0$, which imposes upper bounds on the VEVs as in the previous case. Moreover, the existence of J_R in the physical spectrum brings an extra difficulty that rules out the model once and for all when that \mathbb{Z}_2 symmetry is considered. This difficulty comes from the invisible decay width of the Z gauge boson which receives at least one new contribution, $Z \rightarrow J_R J_I$, and there is no more room for this [43]. Therefore, we can conclude that the model invariant under the \mathbb{Z}_2 symmetry is not consistent with constraints coming from astrophysics and particle physics.

IV. MODEL WITH SOFT \mathbb{Z}_2 -BREAKING TERMS

There is another possibility to be taken into account when the \mathbb{Z}_2 discrete symmetry is considered. This is to introduce some soft \mathbb{Z}_2 -breaking terms in the scalar potential in order to remove the extra accidental symmetries in the Lagrangian, and at the same time leave the model as simple as possible. With that in mind, we are going to explore two terms,

$\mu_4^2 \chi^\dagger \eta$ and $\frac{f}{\sqrt{2}} \epsilon_{ijk} \eta_i \rho_j \chi_k$, that can in principle accomplish that. Let us consider both cases separately by starting from the $\mu_4^2 \chi^\dagger \eta$ term.

A. $\mu_4^2 \chi^\dagger \eta$ term

First, we consider the $\mu_4^2 \chi^\dagger \eta$ soft-breaking term together with the scalar potential allowed by the \mathbb{Z}_2 symmetry. Although this term does not remove any extra symmetry in Table I, its presence slightly changes the form of the NG boson in comparison to the case of the model without it. In some sense that justifies our next analysis. In order to extract conclusions we obtain the physical NG boson for each symmetry breaking pattern. The NG boson looks in each case as follows

$$v_{\eta_1}, v_{\rho_2}, v_{\chi_3} \rightarrow J_1 = N_{J_1}^{-1} \left(v_{\chi_3} \text{Im } \eta_1^0 + \frac{v_{\eta_1} v_{\chi_3}}{v_{\rho_2}} \text{Im } \rho_2^0 + v_{\eta_1} \text{Im } \chi_3^0 \right), \quad (21)$$

$$v_{\eta_1}, v_{\eta_3}, v_{\rho_2}, v_{\chi_3} \rightarrow J_2 = N_{J_2}^{-1} \left(v_{\chi_3} \text{Im } \eta_1^0 + \frac{v_{\eta_1} v_{\chi_3}}{v_{\rho_2}} \text{Im } \rho_2^0 + v_{\eta_1} \text{Im } \chi_3^0 - v_{\eta_3} \text{Im } \chi_1^0 \right), \quad (22)$$

$$v_{\eta_1}, v_{\rho_2}, v_{\chi_1}, v_{\chi_3} \rightarrow J_3 = N_{J_3}^{-1} \left(v_{\chi_3} \text{Im } \eta_1^0 + \frac{v_{\eta_1} v_{\chi_3}}{v_{\rho_2}} \text{Im } \rho_2^0 + v_{\eta_1} \text{Im } \chi_3^0 - v_{\chi_1} \text{Im } \eta_3^0 \right), \quad (23)$$

$$v_{\eta_1}, v_{\eta_3}, v_{\rho_2}, v_{\chi_1}, v_{\chi_3} \rightarrow J_4 = N_{J_4}^{-1} \left(v_{\chi_3} \text{Im } \eta_1^0 - \frac{(v_{\eta_3} v_{\chi_1} - v_{\eta_1} v_{\chi_3})}{v_{\rho_2}} \text{Im } \rho_2^0 + v_{\eta_1} \text{Im } \chi_3^0 - v_{\chi_1} \text{Im } \eta_3^0 - v_{\eta_3} \text{Im } \chi_1^0 \right), \quad (24)$$

where N_{J_i} (normalization factors) are given by $N_{J_i} \equiv \left(C_{\eta_1}^2 + C_{\eta_3}^2 + C_{\rho_2}^2 + C_{\chi_1}^2 + C_{\chi_3}^2 \right)^{1/2}$, and we also have defined $J_i \equiv N_{J_i}^{-1} [C_{\rho_2} \text{Im } \rho_2^0 + C_{\eta_1} \text{Im } \eta_1^0 + C_{\eta_3} \text{Im } \eta_3^0 + C_{\chi_1} \text{Im } \chi_1^0 + C_{\chi_3} \text{Im } \chi_3^0]$.

One can note that the component $\text{Im } \rho_2^0$ in J accounts for a non-zero value of $g_{e\bar{e}J}$, which is generically given by $g_{e\bar{e}J_i} = \frac{\sqrt{2}m_e}{v_{\rho_2}} N_{J_i}^{-1} C_{\rho_2}$. The case $(v_{\eta_1}, v_{\rho_2}, v_{\chi_3})$ is ruled out following discussion after Eq. (18) since we exactly recover the case of Eqs. (14) and (15). For the cases with more than three non-vanishing VEVs, one has to deal with more variables which makes these three scenarios - J_2 , J_3 and J_4 - not easy to exclude using the same strategy followed in Section III. The best we can do with that strategy is set an upper bound on v_{χ_3} . For the case of four VEVs we manage to find $v_{\chi_3} \lesssim 355$ GeV. The case of the five VEVs is more intricate although a similar bound (although not general) is possible

to get. Roughly speaking, these bounds comes from the simultaneous application of $g_{e\bar{e}J}$, $\rho = M_W^2 / (M_Z^2 \cos^2 \theta_W)$ bounds (M_W , M_Z and θ_W are the W boson mass, Z boson mass and the weak mixing angle, respectively [43]).

There is another immediate consequence arising from the J form. Following Eq. (4), and taking into account the expressions of the J NG bosons in Eqs. (21-24), we are led to the conclusion that they are going to interact inevitably with the quarks of the model. It implies that the g_{nnJ} coupling, the interaction of two nucleons with J , is different from zero in this case. Nevertheless, it must be smaller than 10^{-12} , i.e., $|g_{nnJ}| \lesssim g_{nnJ}^{\max} \equiv 10^{-12}$ [15–21]. We have checked that this bound eventually imposes strong constraints on the VEVs and the Yukawa couplings of the model. In general, we have seen that some parameters are forced to have an $\mathcal{O}(10^{-15})$ tuning in order for the limit on g_{nnJ} to be respected. However, we are not going into the details because there is a different line of reasoning that unquestionably rules out this scenario with just the $\mu_4^2 \chi^\dagger \eta$ term. It consists on the observation that the $U(1)_{PQ}$ which generates the NG boson (in this case it is an axion) is actually broken in the electroweak scale. In order to see that, note that the $3U(1)_N + U(1)_{PQ}$ subgroup is not spontaneously broken by any η_i or χ_i VEVs. Thus, the responsible for breaking that subgroup is the ρ VEV which is upper-bounded by the electroweak scale ($V_{SM} \simeq 246$ GeV) since it is mainly responsible for giving mass to all SM leptons in this model. Therefore, the NG boson (axion) would be visible and thus already ruled out. All in all, the soft $\mu_4^2 \chi^\dagger \eta$ breaking term is not capable to get rid of the issues arising from the presence of a NG boson in this model.

B. $\frac{f}{\sqrt{2}} \epsilon_{ijk} \eta_i \rho_j \chi_k$ term

From Table I, we can see that the $\frac{f}{\sqrt{2}} \epsilon_{ijk} \eta_i \rho_j \chi_k$ term removes the $U(1)_{PQ}$ symmetry. As a result there are no physical NG bosons, fact which leaves the model safe regarding the appearance of these massless particles. Therefore, we conclude that this model can be considered with the \mathbb{Z}_2 symmetry provided the soft $\frac{f}{\sqrt{2}} \epsilon_{ijk} \eta_i \rho_j \chi_k$ term is included in the scalar potential.

V. CONCLUSIONS

The presence of physical NG bosons can play a potentially important role in constraining the parameters of a model, and in some cases excludes it. The reason is that any light neutral particle can interact with matter providing an important stellar energy-loss mechanism. With this motivation, we have constrained an appealing version of the $3 - 3 - 1$ model, taking into account the presence of physical NG bosons, which are inconsistent with both astrophysical and particle physics well established results. Specifically, we have considered the $3 - 3 - 1$ model with right-handed neutrinos when a \mathbb{Z}_2 symmetry is imposed. This scenario is conventionally considered in the literature because it greatly simplifies the model. However, that discrete symmetry brings as a consequence the introduction of an extra global symmetry, $U(1)_{PQ}$, which when spontaneously broken by the $v_{\eta_1}, v_{\rho_2}, v_{\chi_3}$ VEVs, introduces an axion in the model. The issue is that imposing simultaneously the bounds on $g_{e\bar{e}J}$ and M_W we found bounds on the decay coupling constant f_a , specifically, $11.5 \text{ keV} \leq f_a \leq v_{SM}$. It is inconsistent with experiments looking for light scalars (or pseudo-scalars) since that window for f_a makes the axion visible. In addition to that, we have considered all the other possibilities for the available VEVs in the model, showing that these are also excluded. The reason is that the appearance of physical NG bosons contributing to the invisible decay width of the Z gauge boson ($Z \rightarrow J_R J_I$) is in conflict with experimental data. All of these scenarios are then ruled out.

We also studied in detail the soft \mathbb{Z}_2 -symmetry breaking case. The two terms allowed by the gauge symmetries are $\mu_4^2 \chi^\dagger \eta$ and $\frac{f}{\sqrt{2}} \epsilon_{ijk} \eta_i \rho_j \chi_k$. In the first case, we found that although the NG bosons have slightly different forms, there are also problems that exclude all of those scenarios. The main reason is that a $3U(1)_N + U(1)_{PQ}$ subgroup remains unbroken after the first step of the spontaneous symmetry breaking. Thus, the decay coupling constant is of the order of the electroweak scale, v_{SM} , implying the same issues as the previous case. Finally, we show that the model can be considered consistent with the \mathbb{Z}_2 symmetry when the $\frac{f}{\sqrt{2}} \epsilon_{ijk} \eta_i \rho_j \chi_k$ term is included in the scalar potential. That term breaks the \mathbb{Z}_2 symmetry softly and removes the extra global symmetry. Thus, no physical NG boson appears at all and the model is safe concerning this issue.

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